The Flow of a Formative Assessment Lesson

Some Formative Assessment Lessons (FALs) focus on content, some focus on problem solving. This document illustrates how a FAL focused on problem solving unfolds and explains the nature of these lessons.

BEFORE THE FORMATIVE ASSESSMENT LESSON

The teacher invites students to work on the problem *Boomerangs* individually for fifteen minutes. It is important to allow students to grapple with the problems posed in *Boomerangs* without assistance, as far as possible.

BOOMERANGS

Phil and Cath make and sell boomerangs for a school event. The money they raise will go to charity.

They plan to make them in two sizes: small and large.

Phil will carve them from wood. The small boomerang takes 2 hours to carve and the large one takes 3 hours to carve. Phil has a total of 24 hours available for carving.

Cath will decorate them. She only has time to decorate 10 boomerangs of either size.

The small boomerang will make $8 for charity. The large boomerang will make $10 for charity.

They want to make as much money for charity as they can. How many small and large boomerangs should they make? How much money will they then make?

These formative assessment lessons were developed by the Shell Centre at the University of Nottingham, England, through a grant from the Gates Foundation.

COMMON CORE STATE STANDARDS

The mathematics of this lesson is central to the Common Core State Standards (CCSS). The standards call upon students to represent constraints by equations and inequalities, and by systems of equations and/or inequalities, and to interpret solutions as viable or non-viable options in a modeling context.

The standards also call upon students to graph solutions to linear inequalities in two variables as a half-plane, and to graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

A FOCUS ON PROBLEM SOLVING

At the heart of this lesson is the task *Boomerangs*. Its main goal is to enable students to develop strategies for solving problems in whose scenarios limited resources must be used to greatest effect. In solving *Boomerangs*, students are called upon to help Phil and Cath maximize profits by optimizing the use of both time and materials.

Assessing students' responses

Teachers collect and analyze their students' responses to *Boomerangs*. The goal here is to identify salient issues and difficulties.

Teachers do not grade their students' work, because providing grades can distract students' attention away from the mathematics that they are intended to learn in the following FAL. Instead, teachers summarize their students' response to *Boomerangs* as a series of questions and comments. These often reflect common issues, and can look like this:

Common issues	Suggested questions and prompts
Difficulty getting started	• What do you know? • What do you need to find out?
Incorrect interpretation of the constraints and variables For example: the student has found the profit for making just one type of boomerang.	• What figures in the task are fixed? • What can you vary? • What is the greatest number of small/large boomerangs they can make?
Unsystematic work For example: The student finds out three or four seemingly unconnected combinations, such as 5 small and 5 large boomerangs, then 10 large, etc.	• Can you organize the numbers of large and small boomerangs made in a systematic way? • What would be sensible values to try? Why? • How can you check that you remember all the constraints?
Poor presentation of work For example: The student presents the work as a series of unexplained numbers and/or calculations, or as a table without headings.	• Would someone unfamiliar with your type of solution be able to understand your work? • Have you explained how you arrived at your answer?
Difficulties in using a graphical approach. Difficulties in using an algebraic approach.	• Would someone unfamiliar with your type of solution be able to understand your work? • How can you check your answer? • How do your answers help you solve the problem?
Correct solution Student needs an extension task.	• Can you now use a different method? For example, a table or graph, or algebra?

ON THE DAY OF THE FORMATIVE ASSESSMENT LESSON

Students work individually on *Boomerangs*. (10 minutes)

Teachers give back the student work and the related comments and questions. Students re-engage with their solutions individually for ten minutes; the goal here is for students to address the issues raised by their teacher's feedback.

> Recall what we were looking at in a previous lesson. What was the task?
>
> I have read your solutions and I have some questions about your work.
>
> I would like you to work on your own for about ten minutes to answer my questions.

Students work collaboratively on *Boomerangs*. (10 minutes)

The teacher then organizes the class into groups of two or three students and gives out a fresh piece of paper to each group.

Teachers invite their students to try the task again, this time combining their ideas.

> Put your own work aside until later in the lesson. I want you to work in groups now.
>
> Your task is to produce a solution together that is better than your individual solutions.

Students work collaboratively to analyze sample student responses to *Boomerangs*. (20 minutes)

After each group has had sufficient time to attempt *Boomerangs* again collaboratively, the teacher gives out a copy of the sample responses of Alex, Danny, Jeremiah, and Tanya. The teacher then asks students to analyze the sample work and write comments on it. Inviting students to analyze other students' work provides the opportunity to evaluate a variety of possible approaches to *Boomerangs*, but without providing a complete solution strategy.

> Imagine you are the teacher and have to assess this work.
>
> Correct the work and write comments about the accuracy and organization of each response.

STUDENTS PUT FEEDBACK TO WORK

Teachers create the opportunity for students to grapple with their initial responses to *Boomerangs* and their teacher's task-specific feedback.

STUDENTS ADDRESS THEIR TEACHER'S TASK-SPECIFIC QUESTIONS

The teacher's task-specific questions enable students to reengage with the mathematics of *Boomerangs*. This feature of the FAL is designed to move students' attention away from simply getting answers and toward learning the mathematics that they need to solve the problem.

THE ROLE OF COLLABORATIVE WORK IN FORMATIVE ASSESSMENT

Collaborative work in mathematics provides students with an opportunity to exchange constructive feedback with their peers by comparing and contrasting different solution paths, evaluating various strategies, and resolving issues through discussion.

STUDENTS ANALYZE STUDENTS' RESPONSES

Another opportunity to grapple with the intrinsic complexities of mathematics is provided when students are given four carefully selected pieces of sample student work. These sample responses, generated by actual students, are selected to expose students to a wider range of solution paths, errors, insights, and approaches that are variously effective or ineffective.

Sample responses to discuss

ALEX

Phil can only make 12 small or 8 large boomerangs in 24 hours

12 small makes $96
8 large makes $80

Cath only has time to make 10, so $96 is impossible.
She could make 10 small boomerangs which will make $80.
So she either makes 8 large or 10 small boomerangs and makes $80

JEREMIAH

Small boomerangs = x
Large boomerangs = y

Time to carve $2x + 3y = 24$ ①
Only 10 can be decorated $x + y = 10$ ②
       $2x + 2y = 20$ ③
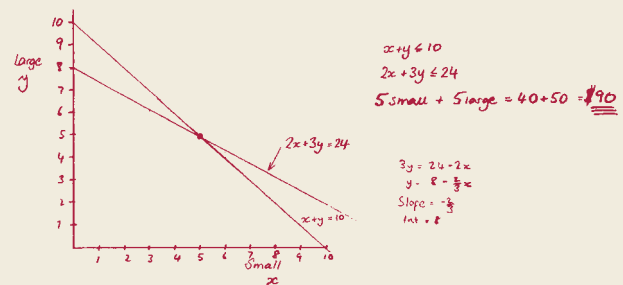
①-③ $y = 4$ $x = 6$

So make 4 large boomerangs
6 small boomerangs

DANNY

No of small s	S × 8	No of large	ℓ × 10	Profit
0	0	8	80	80
1	8	8̸ 7	70	78
2	16	6	60	76
3	24	5	50	74
4	32	5	50	82 ←
5	40	4	40	80
6	48	3	30	78

The most Profit is $82

TANYA

$x + y \leq 10$
$2x + 3y \leq 24$
5 small + 5 large = 40 + 50 = $90

$2x + 3y = 24$
$3y = 24 - 2x$
$y = 8 - \frac{2}{3}x$
Slope = $-\frac{2}{3}$
Int = 8

Students participate in a whole-class discussion: Comparing different approaches. (10 minutes)

Teachers facilitate a whole-class discussion to consider the different approaches used by Alex, Danny, Jeremiah, and Tanya when they solved *Boomerangs*. Here teachers focus the discussion on those parts of the collaborative activities that students found difficult. Teachers ask students to compare the different solution methods.

> Which approach did you like best? Why?
>
> Which approach did you find most difficult to understand?

Students will get to share the strengths and weaknesses of the sample responses when the teacher facilitates a whole-class discussion. They will realize that Alex has considered both constraints, but has not examined different combinations; appreciate both the effectiveness and limitations of Danny's table; understand that Jeremiah's equations really ought to be inequalities; and realize the potential of Tanya's graphical approach.

Students return to *Boomerangs* individually and work to improve their own work. (10 minutes)

Finally, teachers invite students to revise their own work using a different color pen than that of their first solution. If students are satisfied with their work, teachers may ask them to try a different approach to *Boomerangs*. Teachers encourage students to compare their new approach with the first one they tried.

STUDENTS RETURN TO THEIR INITIAL RESPONSES AND PERFECT THEM

In this closing activity, students are given the opportunity to return to their individual responses to *Boomerangs* and perfect them, using all that they have learned throughout the lesson.

High School Algebra I

The following pathways are from the Common Core State Standards for Mathematics Appendix A. The charts show the connections between FALs and high school math subjects.

Units	Includes Standard Clusters	Formative Assessment Lessons	Mathematical Practice Standards
UNIT 1 RELATIONSHIPS BETWEEN QUANTITIES AND REASONING WITH EQUATIONS	• Reason quantitatively and use units to solve problems. • Interpret the structure of expressions. • Create equations that describe numbers or relationships. • Understand solving equations as a process of reasoning and explain the reasoning. • Solve equations and inequalities in one variable.	Changing the Subject of a Formula Increasing and Decreasing Quantities by a Percent Estimations and Approximations	Make sense of problems and persevere in solving them.
UNIT 2 LINEAR AND EXPONENTIAL RELATIONSHIPS	• Extend the properties of exponents to rational exponents. • Solve systems of equations. • Represent and solve equations and inequalities graphically. • Understand the concept of a function and use function notation. • Interpret functions that arise in applications in terms of a context. • Analyze functions using different representations. • Build a function that models a relationship between two quantities. • Build new functions from existing functions. • Construct and compare linear, quadratic, and exponential models and solve problems. • Interpret expressions for functions in terms of the situation they model.	Generalizing Patterns: Table Tiles Interpreting Distance–Time Graphs Solving Linear Equations in Two Variables Solving Optimization Problems Interpreting Algebraic Expressions Equations and Identities Inequalities	Reason abstractly and quantitatively. Construct viable arguments and critique the reasoning of others. Model with mathematics.
UNIT 3 DESCRIPTIVE STATISTICS	• Summarize, represent, and interpret data on a single count or measurement variable. • Summarize, represent, and interpret data on two categorical and quantitative variables. • Interpret linear models.	Interpreting Statistics	Use appropriate tools strategically.
UNIT 4 EXPRESSIONS AND EQUATIONS	• Interpret the structure of expressions. • Write expressions in equivalent forms to solve problems. • Perform arithmetic operations on polynomials. • Create equations that describe numbers or relationships. • Solve equations and inequalities in one variable. • Solve systems of equations.	Linear Equations from Situations	Look for and make use of structure.
UNIT 5 QUADRATIC FUNCTIONS AND MODELING	• Use properties of rational and irrational numbers. • Interpret functions that arise in applications in terms of a context. • Analyze functions using different representations. • Build a function that models a relationship between two quantities. • Build new functions from existing functions. • Construct and compare linear, quadratic, and exponential models and solve problems.	Forming Quadratics	Attend to precision. Look for and express regularity in repeated reasoning.

Geometry

Units	Includes Standard Clusters	Formative Assessment Lessons	Mathematical Practice Standards
UNIT 1 CONGRUENCE, PROOF, AND CONSTRUCTIONS	• Experiment with transformations in the plane. • Understand congruence in terms of rigid motions. • Prove geometric theorems. • Make geometric constructions.	Transformations	Make sense of problems and persevere in solving them.
UNIT 2 SIMILARITY, PROOF, AND TRIGONOMETRY	• Understand similarity in terms of similarity transformations. • Prove theorems involving similarity. • Define trigonometric ratios and solve problems involving right triangles. • Apply geometric concepts in modeling situations. • Apply trigonometry to general triangles.	Solving Pentagon Problems Evaluating Statements about Length and Area	Reason abstractly and quantitatively. Construct viable arguments and critique the reasoning of others.
UNIT 3 EXTENDING TO THREE DIMENSIONS	• Explain volume formulas and use them to solve problems. • Visualize the relation between two-dimensional and three-dimensional objects. • Apply geometric concepts in modeling situations.	Solving Real-World Measurement Problems Solving Enlargement Problems	Model with mathematics.
UNIT 4 CONNECTING ALGEBRA AND GEOMETRY THROUGH COORDINATES	• Use coordinates to prove simple geometric theorems algebraically. • Translate between the geometric description and the equation for a conic section.		Use appropriate tools strategically.
UNIT 5 CIRCLES WITH AND WITHOUT COORDINATES	• Understand and apply theorems about circles. • Find arc lengths and areas of sectors of circles. • Translate between the geometric description and the equation for a conic section. • Use coordinates to prove simple geometric theorems algebraically. • Apply geometric concepts in modeling situations.	Solving Circle, Triangle, and Square Problems Modeling with Geometry: Rolling Cups	Look for and make use of structure. Attend to precision.
UNIT 6 APPLICATIONS OF PROBABILITY	• Understand independence and conditional probability and use them to interpret data. • Use the rules of probability to compute probabilities of compound events in a uniform probability model. • Use probability to evaluate outcomes of decisions.	Evaluating Statements about Probability	Look for and express regularity in repeated reasoning.

Units	Includes Standard Clusters	Formative Assessment Lessons	Mathematical Practice Standards
UNIT 1 RELATIONSHIPS BETWEEN QUANTITIES	• Reason quantitatively and use units to solve problems. • Interpret the structure of expressions. • Create equations that describe numbers or relationships.	Increasing and Decreasing Quantities by a Percent	Make sense of problems and persevere in solving them.
UNIT 2 LINEAR AND EXPONENTIAL RELATIONSHIPS	• Represent and solve equations and inequalities graphically. • Understand the concept of a function and use function notation. • Interpret functions that arise in applications in terms of a context. • Analyze functions using different representations. • Build a function that models a relationship between two quantities. • Build new functions from existing functions. • Construct and compare linear, quadratic, and exponential models and solve problems. • Interpret expressions for functions in terms of the situation they model.	Generalizing Patterns: Table Tiles Interpreting Distance–Time Graphs Solving Linear Equations in Two Variables Solving Optimization Problems Interpreting Algebraic Expressions Equations and Identities Inequalities	Reason abstractly and quantitatively. Construct viable arguments and critique the reasoning of others.
UNIT 3 REASONING WITH EQUATIONS	• Understand solving equations as a process of reasoning and explain the reasoning. • Solve equations and inequalities in one variable. • Solve systems of equations.	Changing the Subject of a Formula	Model with mathematics.
UNIT 4 DESCRIPTIVE STATISTICS	• Summarize, represent, and interpret data on a single count or measurement variable. • Summarize, represent, and interpret data on two categorical and quantitative variables. • Interpret linear models.	Interpreting Statistics	Use appropriate tools strategically.
UNIT 5 CONGRUENCE, PROOF, AND CONSTRUCTIONS	• Experiment with transformations in the plane. • Understand congruence in terms of rigid motions. • Make geometric constructions.	Transformations	Look for and make use of structure. Attend to precision.
UNIT 6 CONNECTING ALGEBRA AND GEOMETRY THROUGH COORDINATES	• Use coordinates to prove simple geometric theorems algebraically.		Look for and express regularity in repeated reasoning.

Units	Includes Standard Clusters	Formative Assessment Lessons	Mathematical Practice Standards
UNIT 1 EXTENDING THE NUMBER SYSTEM	• Extend the properties of exponents to rational exponents. • Use properties of rational and irrational numbers. • Perform arithmetic operations with complex numbers. • Perform arithmetic operations on polynomials.		Make sense of problems and persevere in solving them.
UNIT 2 QUADRATIC FUNCTIONS AND MODELING	• Interpret functions that arise in applications in terms of a context. • Analyze functions using different representations. • Build a function that models a relationship between two quantities. • Build new functions from existing functions. • Construct and compare linear, quadratic, and exponential models and solve problems.	Forming Quadratics	Reason abstractly and quantitatively.
UNIT 3 EXPRESSIONS AND EQUATIONS	• Interpret the structure of expressions. • Write expressions in equivalent forms to solve problems. • Create equations that describe numbers or relationships. • Solve equations and inequalities in one variable. • Use complex numbers in polynomial identities and equations. • Solve systems of equations.	Linear Equations from Situations	Construct viable arguments and critique the reasoning of others. Model with mathematics.
UNIT 4 APPLICATIONS OF PROBABILITY	• Understand independence and conditional probability and use them to interpret data. • Use the rules of probability to compute probabilities of compound events in a uniform probability model. • Use probability to evaluate outcomes of decisions.	Evaluating Statements about Probability	Use appropriate tools strategically.
UNIT 5 SIMILARITY, RIGHT TRIANGLE TRIGONOMETRY, AND PROOF	• Understand similarity in terms of similarity transformations. • Prove geometric theorems. • Prove theorems involving similarity. • Use coordinates to prove simple geometric theorems algebraically. • Define trigonometric ratios and solve problems involving right triangles. • Prove and apply trigonometric identities.	Solving Pentagon Problems Evaluating Statements about Length and Area	Look for and make use of structure. Attend to precision.
UNIT 6 CIRCLES WITH AND WITHOUT COORDINATES	• Understand and apply theorems about circles. • Find arc lengths and areas of sectors of circles. • Translate between the geometric description and the equation for a conic section. • Use coordinates to prove simple geometric theorem algebraically. • Explain volume formulas and use them to solve problems.	Solving Circle, Triangle, and Square Problems Modeling with Geometry: Rolling Cups	Look for and express regularity in repeated reasoning.